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## Cross-correlations of conserved charges from the lattice

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**Abstract**

We present a lattice calculation on the cross-correlations of conserved charges (baryon number, electric charge and strangeness) near the transition temperature. We extrapolate to small baryo-chemical potentials, and thus we cover typical STAR energies. We confront our finding to the latest STAR data set on cross-correlations. In this work we present continuum lattice results with resolution up to  $N_t = 16$ .

**Keywords:** lattice QCD, cross-correlations, phase diagram, finite density

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**1. Introduction**

Correlations of conserved charges are important observables for the finite-density investigations. In this work we focus on the off-diagonal combinations. One possible way to extend lattice results to finite density is to perform Taylor expansions of the thermodynamic observables around chemical potential  $\mu_B = 0$  [1, 2, 3, 4, 5]: fluctuations of conserved charges are directly related to the Taylor expansion coefficients of such observables. They allow for a comparison between theoretical and experimental results to extract the chemical freeze-out temperature  $T_f$  and chemical potential  $\mu_{Bf}$  as functions of the collision energy [6, 7, 8, 9]. The higher order fluctuations are also an important signature for the critical endpoint, as they give access to the correlation length [3, 10, 11].

In this work we use analytical continuation from imaginary chemical potential [12, 13, 14, 15, 16]. It agrees well with the results of the Taylor expansion as shown for the transition temperature [17].

We simulate the lower-order fluctuations at imaginary chemical potential and extract the higher order fluctuations as derivatives of the lower order ones at  $\mu_B = 0$ . This method has been successfully used in the past and proved to lead to a more precise determination of the higher order fluctuations, compared to their direct calculation [18, 19, 17].

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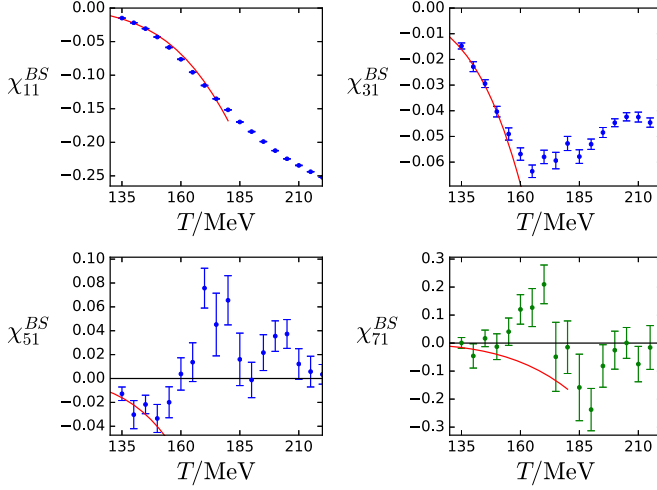


Fig. 1. Results for  $\chi_{11}^{BS}$ ,  $\chi_{31}^{BS}$ ,  $\chi_{51}^{BS}$  and an estimate for  $\chi_{71}^{BS}$  on a  $N_t = 12$  lattice as functions of the temperature, obtained from the single-temperature analysis. We plot  $\chi_{71}^{BS}$  in green to point out that its determination is guided by a prior, which is linked to  $\chi_{31}^{BS}$ . The red curve in each panel corresponds to the Hadron Resonance Gas (HRG) model result. [20]

## 2. Cross-correlations on an $N_t = 12$ -lattice

We first present an analysis with high precision on an  $N_t = 12$  lattice. A more detailed description as well as precise information on the lattice set-up can be found in ref. [20, 21]. In the following we use the notation  $\chi_{i,j,k}^{B,Q,S} = \frac{\partial^{i+j+k}(p/T^4)}{(\partial\hat{\mu}_B)^i(\partial\hat{\mu}_Q)^j(\partial\hat{\mu}_S)^k}$ , with  $\hat{\mu} = \mu/T$ . We make the ansatz

$$\chi_{01}^{BS}(\hat{\mu}_B) = \chi_{11}^{BS}\hat{\mu}_B + \frac{1}{3!}\chi_{31}^{BS}\hat{\mu}_B^3 + \frac{1}{5!}\chi_{51}^{BS}\hat{\mu}_B^5 + \frac{1}{7!}\chi_{71}^{BS}\hat{\mu}_B^7 + \frac{1}{9!}\chi_{91}^{BS}\hat{\mu}_B^9 \quad (1)$$

where  $\frac{\chi_{31}^{BS}}{\chi_{71}^{BS}}$  and  $\frac{\chi_{51}^{BS}}{\chi_{91}^{BS}}$  are constrained by a prior, normally distributed with  $\mu = -1.25$  and  $\sigma = 2.75$  and the independent fit parameters are  $\chi_{11}^{BS}$ ,  $\chi_{31}^{BS}$  and  $\chi_{51}^{BS}$ . The results which we obtain from a fully correlated fit to this ansatz and its first three derivatives  $\chi_{11}^{BS}(\hat{\mu}_B)$ ,  $\chi_{31}^{BS}(\hat{\mu}_B)$  and  $\chi_{51}^{BS}(\hat{\mu}_B)$  are presented in fig. 1. To connect to experimental results, we calculate the ratio of the cumulants of the net-baryon number distribution as functions of temperature and chemical potential by means of their Taylor expansion in powers of  $\mu_B/T$ . This is possible by combining different diagonal and non-diagonal fluctuations to obtain a result at the strangeness neutral point and with  $\langle n_Q \rangle = 0.4\langle n_B \rangle$ . Our results for  $S_B\sigma_B^3/M_B$  and  $\kappa_B/\sigma_B^2$  are shown in fig. 2. Here  $M_B$  is the mean,  $\sigma_B^2$  is the variance,  $S_B$  is the skewness and  $\kappa_B$  the kurtosis of the the baryon number distribution.

## 3. Cross-correlations in the continuum

Now we will present our preliminary results on the cross-correlations in the continuum. The curves shown in fig. 3 are not final, as they do not yet include a full analysis of the systematic error. To extrapolate to the continuum we need to incorporate the temperature and the  $1/N_t^2$  dependence of our data in this fit ansatz. We expect our results for  $\chi(T)$  to lie on a smooth curve. We implement this information by fitting the results with a spline in the temperature direction. For the  $1/N_t^2$  we fit a linear function through the data from lattices with the sizes  $32^3 \times 10$ ,  $40^3 \times 10$ ,  $40^3 \times 12$ ,  $48^3 \times 12$ ,  $48^3 \times 16$  and  $64^3 \times 16$ . Our whole analysis is done in one combined fit. Therefore now the fit parameter in ansatz similar to eq. (1) become functions of  $T$  and  $N_t$ .

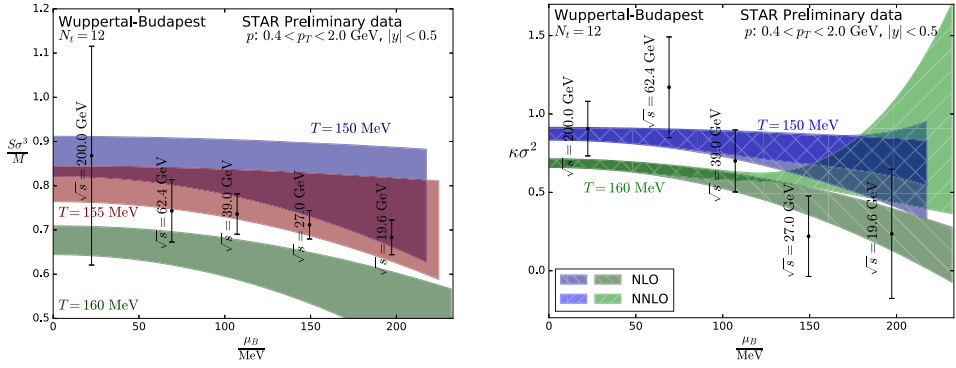


Fig. 2.  $S_B \sigma_B^3 / M_B$  (left panel) and  $\kappa_B \sigma_B^2$  (right panel) extrapolated to finite chemical potential. The left panel is extrapolated up to  $O(\mu_B^2)$ . In the right panel, the darker bands correspond to the extrapolation up to  $O(\mu_B^2)$ , whereas the lighter bands also include the  $O(\mu_B^4)$  term. [20]

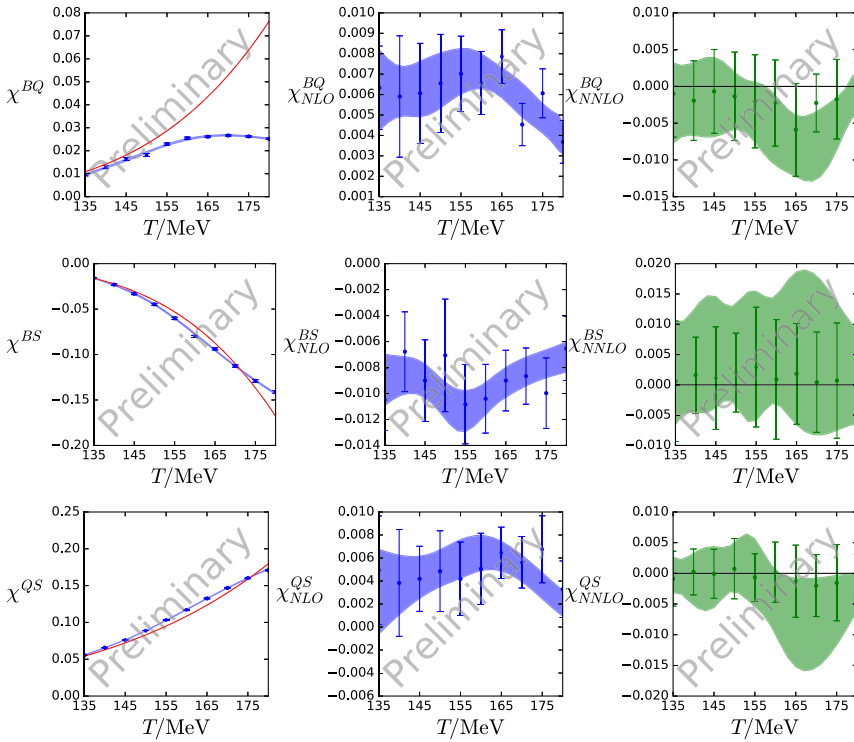


Fig. 3. Preliminary results for the cross-correlations in the continuum. The NNLO contribution is plotted in green, as it is again constrained by a prior. The red curves correspond to the HRG model results.

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